



Neutrino 2012: Outlook - theory

A. Yu. Smirnov^a

^a*International Center for Theoretical Physics, Trieste, Italy*

Abstract

Ongoing developments in theory and phenomenology are related to the measured large value of 1-3 mixing and indications of significant deviation of the 2-3 mixing from maximal one. “Race” for the mass hierarchy has started and there is good chance that multi-megaton scale atmospheric neutrino detectors with low threshold (e.g. PINGU) will establish the type of hierarchy. Two IceCube candidates of the PeV cosmic neutrinos if confirmed, is the beginning of new era of high energy neutrino astronomy. Accumulation of data on solar neutrinos (energy spectrum, D-N asymmetry, value of Δm_{21}^2) may uncover some new physics. The Tri-bimaximal mixing is disfavored and the existing discrete symmetry paradigm may change. The confirmed QLC prediction, $\theta_{13} \approx \theta_C / \sqrt{2}$, testifies for GUT, seesaw and some symmetry at very high scales. However, the same value of 1-3 mixing can be obtained in various ways which have different implications. The situation in lepton sector changes from special (with specific neutrino symmetries, *etc.*) to normal, closer to that in the quark sector. Sterile neutrinos are challenge for neutrino physics but also opportunity with many interesting phenomenological consequences. Further studies of possible connections between neutrinos and the dark sector of the Universe may lead to breakthrough both in particle physics and cosmology.

Keywords: Neutrinos, masses, mixing

1. Introduction

Kyoto ¹ is a special place, full of spiritual motions, enlightening and insights (see e.g., the poster of the conference), the place to meditate and look for the signs of future.

The standard 3 neutrino framework is the reference point. Global view from the global fits can be summarized in the following way:

1. There are three neutrinos with two salient and probably related features: (i) smallness of the neutrino masses, (ii) peculiar pattern of the lepton mixing which substantially differs from the quark mixing pattern.

2. The nature neutrino mass is among still missing elements. It has two aspects: (1) Dirac versus Majorana and (2) “hard” versus “soft”. The usual hard masses are generated at the electroweak and higher mass scales.

The soft (environment dependent) contributions to the mass, and in general, to the dispersion relation ² are due to some new interactions of neutrinos with medium composed of known or new fields.

3. The main challenge for this picture is possible existence of sterile neutrinos. In fact, introduction of “steriles” with properties required by the LSND, Mini-BooNE, reactor and Gallium anomalies is not a small perturbation of the standard framework.

Results of the global fit of the oscillation data updated after the conference are given in [1]. The key points, which have far going implications for theory, are

- (i) Rather large value of the 1-3 mixing angle (fig. 1).
- (ii) Evidence of substantial deviation of the 2-3 mixing from maximal one.
- (iii) Indication (from the global fit) of preferable value of the CP-phase $\delta \approx 1.1\pi$.

Measurement of the 1-3 mixing has strong impact on

¹Talk given at the XXV International Conference on Neutrino Physics and Astrophysics, June 3 - 9, 2012, Kyoto, Japan

²Recall that it is the dispersion relation which is probed in oscillations.

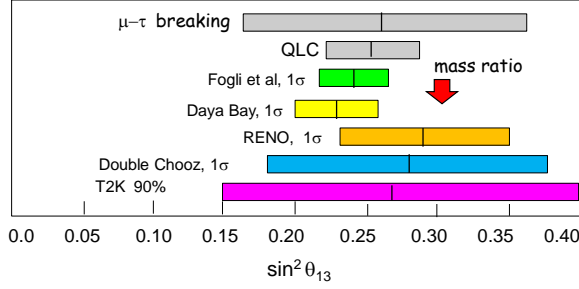


Figure 1: Determination of the 1-3 mixing. Shown are the results from Daya-Bay experiment, T2K and global fit. The upper bands correspond to the QLC predictions and prediction from $\nu_\mu - \nu_\tau$ symmetry breaking. The arrow indicates the value of mass ratio $\Delta m_{21}^2 / \Delta m_{31}^2$.

many areas of neutrino physics [2]. It fixes the resonance structure of oscillograms of the Earth at high energies > 1 GeV, and therefore determines effects for accelerator and atmospheric neutrino fluxes in this range. The 1-3 mixing is the key parameter for supernova neutrino conversion. It also affects the solar neutrinos and predictions for the double beta decay. The 1-3 mixing is the door to determination of the mass hierarchy and CP-violation. It has far-going theoretical implications.

2. Race for the mass hierarchy

Determination of the neutrino mass hierarchy will probably be the next step in reconstruction of neutrino mass and flavor spectrum. Physical consequences of the hierarchy change can be seen immediately from the mass and flavor spectra. In the double beta decay the effective mass $m_{ee}^{IH} > m_{ee}^{NH}$ for the lightest mass $m_{lightest} < 0.05$ eV. In the case of hierarchical spectrum the sum of masses (probed in Cosmology) is two time larger for inverted hierarchy [3]:

$$\sum_i m_i^{IH} = 2 \sum_i m_i^{NH} \approx 2 \sqrt{\Delta m_{atm}^2}. \quad (1)$$

Recall that the mass eigenstates can be marked by their ν_e content (*i.e.*, amount of admixture of the electron flavor). ν_1 has the largest ν_e admixture, $|U_{e1}|^2$, ν_2 - about 2 times smaller, $|U_{e2}|^2$, and ν_3 - the smallest one $|U_{e3}|^2$. Consequently, vacuum oscillations due to the 3-1 mass splitting will have 2 times larger amplitude: $D_{31} = 4|U_{e3}|^2|U_{e1}|^2$, than the amplitude due to the 3-2 mass splitting, $D_{32} = 4|U_{e3}|^2|U_{e2}|^2$. In the case of NH the frequencies of oscillations, $\omega_{ij} \equiv \Delta m_{ij}^2 / 2E$, obey inequality $\omega_{31} > \omega_{32}$, whereas in the case of inverted hierarchy: $\omega_{31} < \omega_{32}$. Therefore spectral analysis of the

energy distribution of the electron (anti)neutrino events in the detector should reveal two peaks with frequencies ω_{31} and ω_{32} . In the case of NH lower frequency will have smaller amplitude, whereas for IH the lower frequency will have larger amplitude [4]. Due to finite energy resolution two peaks merge, and the problem will be to establish whether the shoulder (smaller peak) is on the left or the right hand side of the main peak.

Matter effect makes the ν_e flavor heavier and therefore changes two (effective) mass spectra in matter (for NH and IH) differently. The mixing in matter and resonance condition are determined by combination $|\Delta m_{31}^2 \cos 2\theta_{13} - 2EV|$. The change of sign of Δm_{31}^2 is equivalent to $V \rightarrow -V$, and the latter is realized by transition from neutrino to antineutrino. Thus, in the 2ν -case $NH \leftrightarrow IH$ is equivalent to $\nu \leftrightarrow \bar{\nu}$. In particular, the resonance (resonance enhancement of the $\nu_e \rightarrow \nu_{\mu,\tau}$ oscillations) occurs in the neutrino channel for NH and in the antineutrino channel for IH.

Summarizing, there are three different approaches to determine the mass hierarchy:

1. Explore matter effects on the 1-3 mixing. This will be possible studying the atmospheric neutrino fluxes with magnetized detectors, *e.g.* ICAL at INO [5], or with huge atmospheric neutrino detectors, *e.g.*, PINGU. Another possibility is the LBL experiments: from NOvA [6] to “ultimate” LBL proposal, Fermilab - PINGU [7] in which neutrinos will cross the core of the Earth and therefore will undergo the parametric enhancement of oscillations.

Supernova neutrinos are sensitive to the type of mass hierarchy via influence of matter of a star and the Earth on the 1-3 and 1-2 mixings. Various effects can be used [8]:

- a). Strong suppression of the neutronization peak in the case of NH: since $\nu_e \rightarrow \nu_3$, the survival probability $P = \sin^2 \theta_{13} \approx 0.02$, as compared to $P = \sin^2 \theta_{12} \approx 0.31$ in the case of IH.

- b). Modification of the spectra of electron (anti) neutrinos at the accretion and cooling phases: the spectra become two-component, that is, mixture of the original ν_e and ν_μ spectra (and similarly $\bar{\nu}_e$ spectra). Precise composition depends on mass hierarchy.

- c). Oscillatory modulation of the energy spectra due to the Earth matter effect should be seen in the antineutrino channel for NH, and in the neutrino channel for IH. So, if the Earth matter effect is observed for $\bar{\nu}_e$, the NH is established.

The problem here is that the difference of fluxes of the electron and non-electron antineutrinos, and consequently, the oscillation effects in the antineutrino channel, which is most suitable for detection, are small. Fur-

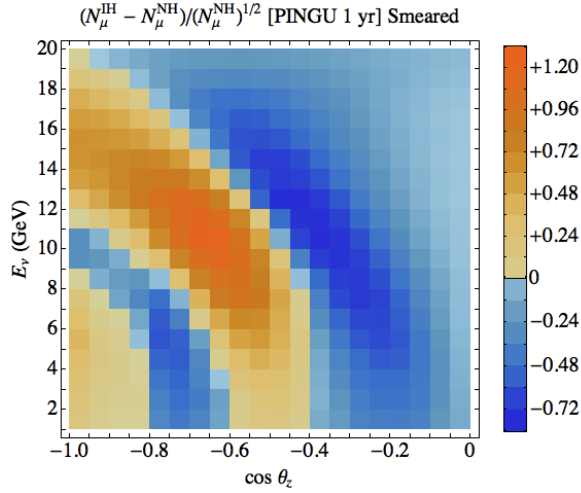


Figure 2: Statistical significance of the determination of the mass hierarchy. The accuracies of the neutrino energy and angle reconstructions are taken to be $\sigma_E = 0.2E_\nu$ and $\sigma_\theta = \sqrt{m_p}/E_\nu$. From [10].

thermore, the collective oscillations in inner parts of a star may complicate the signatures.

2. Precise measurements of Δm^2 (spectral analysis of the oscillation effects discussed above) with the reactor antineutrinos. This, however, put very serious experimental requirements.

3. Study of observables sensitive to the absolute mass scale: the cosmological data and the neutrinoless double beta decay.

Interestingly, existence of sterile neutrinos with the eV scale mass opens up another possibility to establish the hierarchy among active neutrinos: in this case the active-sterile neutrino oscillations in matter of the Earth depend on the hierarchy [9].

It may happen that next advance in the field will be due to huge atmospheric neutrino detectors (HAND's): tenth-megaton mass scale ice (water) cherenkov detectors with relatively low, \sim few GeV, energy threshold. These detectors will collect about 10^5 events/year in the range 2 - 20 GeV which covers the 1-3 resonance region. To determine the hierarchy it is enough to reconstruct the neutrino energy and direction with rather modest accuracy and make certain kinematical (E -zenith angle) selection of events to avoid strong averaging. Fig. 2 illustrate sensitivity to the mass hierarchy: shown is the binned distribution of the quantity $(N_\mu^{IH} - N_\mu^{NH})/\sqrt{N_\mu^{NH}}$, which reflects the statistical significance of the determination in the $E_\nu - \cos \theta_z$ plane for the ν_μ events. Here N_μ^{NH} (N_μ^{IH}) are the number of

events for the normal (inverted) hierarchy [10]. This could be the fastest and the cheapest way to establish the hierarchy, although some instrumental (technological) developments may be needed. HAND's are also sensitive to Δm_{32}^2 and θ_{23} . Once the hierarchy is established one can address the issue of CP-violation searches with HAND's.

3. Other missing elements

3.1. Nature of neutrino mass

With EXO-200 result on the double beta decay [11], the tests of the Heidelberg-Moscow claim [12] enter critical phase. The negative EXO-200 result, $m_{ee} < 0.14 - 0.38$ eV, could be in agreement with the H-M claim only for the smallest possible values of the nuclear matrix elements [11]. Further increase of statistics and therefore improvements of the bound are expected soon. GERDA will publish their first results in 2013 [13]. Determination of the 1-3 mixing does not modify substantially the predicted regions for m_{ee} in the standard 3ν framework [14], since θ_{13} is close to maximal possible value which has been taken in computations of the regions before the θ_{13} determination.

Recent analysis of the cosmological data which includes WMAP 7 years data and new determinations of Hubble constant gives bounds on the sum of neutrino masses at the level (0.3 - 0.4) eV (95% C.L.) [15]. Future progress will be related to the next year Planck release of cosmological data. Although it is not clear whether this will improve the bound on $\sum_i m_i$ [15].

3.2. Deviation of the 2-3 mixing from maximal

Recent data indicate significant deviation of the 2-3 mixing from maximal:

$$d_{23} \equiv 0.5 - \sin^2 \theta_{23} \sim 0.1, \quad (2)$$

see fig. 3. In particular, direct measurements of $\sin^2 \theta_{23}$ by MINOS show the deviation [16]. For the first time analysis of the atmospheric neutrino data by the SK collaboration shows the deviation (with quadrant which depends on the mass hierarchy). Global analysis [1] gives the strongest deviation. Recall that sensitivity to the deviation follows from the atmospheric neutrino data where the fluxes of electron neutrinos $F_e/F_e^0 - 1$ are proportional to $(rc_{23}^2 - 1)P_{e2}$ and $(rs_{23}^2 - 1)P_{e3}$ for sub- and multi-GeV events correspondingly. Since $r \equiv F_\mu^0/F_e^0 \approx 2$ at low energies, the excess of e-like events in the sub-GeV range is proportional to d_{23} .

The appearance probability $P_{\mu e} \propto \sin^2 \theta_{23} \sin^2 2\theta_{13}$. So, after precise determination of the 1-3 mixing with

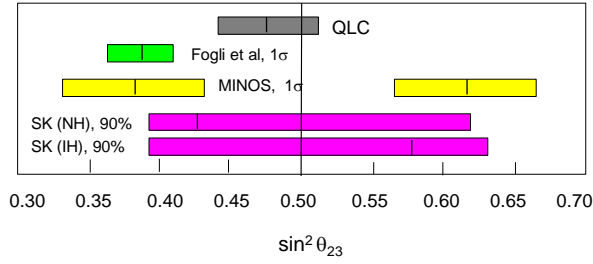


Figure 3: Determination of the 2-3 mixing. Shown are results from MINOS experiment, Atmospheric neutrino studies and global oscillation fit. The arrow indicates the QLC prediction.

reactors, T2K and MINOS results can be used to measure $\sin^2 \theta_{23}$, and it is here, where sensitivity of LBL experiments to the 2-3 mixing comes from. Disappearance experiments are sensitive to $P_{\mu\mu} \propto \sin^2 2\theta_{23}$ (T2K, MINOS). In future HAND's (PINGU [17]), may have good sensitivity to the deviation.

The deviation is the key probe of the underlying physics: it (i) reflects violation of the $\nu_\mu - \nu_\tau$ symmetry as well as symmetry behind the TBM mixing, (ii) is probably connected to the non-zero 1-3 mixing; (iii) is important for the quark-lepton complementarity for which in the lowest order the deviation is small: $\theta_{23} \sim \pi/2 - V_{cb}$.

3.3. CP-phase: measurements and predictions

For both mass hierarchies the global fit gives first glimpses of value of the CP-phase: $\delta_{CP} \sim 1.1\pi$ with 1σ range $(0.4 - 1.2)\pi$, but at 2σ level any value of the phase is allowed [1]. The sensitivity to δ_{CP} comes mainly from the atmospheric neutrino data: the excess of the sub-GeV e-like events. The predicted excess can be enhanced by interference term for $\cos \delta_{CP} = -1$ [1]. Future measurements will be based on comparison of effects in the ν and $\bar{\nu}$ channels (neutrino-antineutrino asymmetry) or on measurements of dependence of the oscillation probabilities in the wide energy range. The third possibility is reconstruction of the unitarity triangle which requires measurements of the depths of neutrino oscillations due to the solar and atmospheric mass splittings in the $\nu_\mu - \nu_\mu$ survival probability [18].

Now it is time to make predictions for δ_{CP} . In fact, in various contexts the maximal violation value, $\delta_{CP} = \pi/2$, often appears (see, e.g., [19]). The doubt is that there is no really convincing theory for the phase in the quark sector. How then the prediction can be made in the lepton sector where more things (e.g. see-saw structure) are involved?

4. Phenomenology

During the meeting the Venus has crossed the solar disc. That was a sign to look at the solar neutrinos again. And indeed, something interesting happens here. Still no upturn of the energy spectrum of electrons has been found at low energies. The upturn expected according to LMA MSW is disfavored at $(1.1 - 1.9)\sigma$ level [20]. By itself this is not very significant, however, no one other experiment has detected the upturn. Furthermore, SNO [21], BOREXINO [22], and KamLAND (solar) [23] indicate that spectrum turns down below 6 MeV. Also indirectly, the Homestake low rate supports this. On the other hand measurements of the Be- and pep- neutrino fluxes by BOREXINO are in a good agreement with the LMA MSW prediction. This means (after BOREXINO) that something non-standard happens in the range $(2 - 7)$ MeV. One possibility is a very light sterile neutrino with $\Delta m_{14}^2 = (1 - 2) 10^{-5} \text{ eV}^2$, which mixes weakly, $\sin^2 2\alpha = (1 - 3) 10^{-4}$, with ν_e [24]. For hierarchical spectrum the corresponding mass equals $m_4 \sim (3 - 4) 10^{-3} \text{ eV}$. It may appear as a combination M^2/M_{Pl} with $M = (2 - 3) \text{ TeV}$ which in turn, implies new physics at the terrascale. If the sterile neutrino mixes in the state ν_3 with relatively large mixing $\sin^2 2\beta \approx 0.1$, an additional radiation in the Universe, ΔN_ν upto 0.9, can be generated.

Another possibility is existence of the non-standard interactions which can modify the energy dependence of conversion in the intermediate energy region [25].

SK reported increasing tension (1.3σ now) between Δm_{21}^2 measured by KamLAND and extracted from the analysis of solar neutrinos. In fact, this can be related to the absence of upturn. SK sees the Day-Night asymmetry of signal at 2.3σ level which is a bit larger than LMA MSW prediction. This also implies larger Δm_{21}^2 .

Solar abundance problem is still unresolved. There is degeneracy of effects of metallicity and opacity: helioseismology can not disentangle them [26]. Measurements of the CNO neutrino fluxes may help. These problems will be addressed by SNO+ [27] and in future by new high statistics and high precision experiments HyperKamiokande, LENA, MICA [28].

Era of oscillation physics with huge atmospheric neutrino detectors has begun. ANTARES reported observation of the ν_μ oscillations with energy threshold $E \sim 20 \text{ GeV}$ at 2.7σ level [29]. DeepCore observes oscillation effect at energies $10 - 100 \text{ GeV}$ at about 5σ . At the same time IceCube does not see oscillations at higher energies $E > 100 \text{ GeV}$ in agreement with standard oscillation predictions. This is important test of theory of

oscillations. It gives bounds on non-standard interactions, Lorentz violation, *etc.*.

Supernova neutrinos: with measured value of the 1-3 mixing the level crossing in the H- (high density) resonance is highly adiabatic. This rejects many possibilities for flavor evolution, and picture of conversion becomes very simple. Adiabaticity is broken in shock wave fronts and therefore the shock waves effects should be observable.

This picture can be affected by the collective oscillation effects which happen in an inner regions of a star [30]. The effects are more important for IH and should be realized during phases when neutrino density becomes larger than usual density. This, in principle, opens up another possibility to establish the mass hierarchy as well as to probe internal structure and physical conditions of collapsing stars.

It was uncovered recently that the collective neutrino Hamiltonian is similar to the BCS pairing Hamiltonian describing superconductivity [31]. This analogy allows one to write down the constants of motion for each individual neutrino mode (characterized by momentum p) [31]. The invariants are important tool to understand various collective effects, in particular spectral splits, and to study stability of the collective evolution.

Recent developments were related to consideration of collective oscillations beyond one dimension where the multi-angle effects become important [32]. Neutrinos arriving at a given space-time point from different directions (multi-angle effect) acquire different phases due to usual matter potential. This leads to decoherence and suppression of the collective phenomena.

Summary of experimental situation with cosmic neutrinos is very simple: Auger see “0” events [33] and IceCube sky is still dark [34], actually almost dark: IceCube has reported (in rather modest way) observation of two candidates - cascades in the 1-10 PeV energy range [35]. This unexpected result can be beginning of new era in the field. Neutrinos can be from the diffuse cosmogenic flux formed during bright phase of the Universe.

Zero Auger result is in agreement with expectations and therefore not dramatic. In contrast, null IceCube results have important implications. Strong bound on the neutrino flux associated to gamma ray bursts, seriously restricts models of GRB as sources of cosmic rays. The results have important implications in view of the neutrino - gamma - CR connections.

5. From special to normal?

In view of relatively large $\sin^2 \theta_{13}$, and some evidences of large deviation d_{23} , the key question is “Symmetry or no symmetry?”

Data further indicate departure from the TBM mixing and violation of the $\nu_\mu - \nu_\tau$ symmetry. This can be illustrated by the best fit values of elements of the third column of the mixing matrix: $\|U_{\alpha 3}\| = (0.15, 0.6, 0.8)$ instead of $(0.0, 0.71, 0.71)$ of TBM. Symmetry relations between the elements of mass matrix are broken substantially: E.g. in the case of NH the mass ratios equal $m_{\mu\mu}/m_{\tau\tau} = 0.56$, and $|m_{e\mu}/m_{e\tau}| = 0 \div \infty$, instead of 1.

The dominant line of thoughts and efforts during last 10 years was that certain residual symmetries are behind the approximate TBM mixing pattern [36]:

1. Mixing appears as a result of different ways of the original flavor symmetry, G_f , breaking in the neutrino and charged lepton (Yukawa) sectors.
2. Symmetry is broken partially in each sector and residual symmetries are different for neutrinos, G_ν , and charged leptons, G_l . The most popular flavor groups are A_4 , S_4 , T' , $\Delta(27)$, T_7 .
3. This difference of symmetry breakings (difference of the flavor properties of the neutrino and charged lepton mass terms) is related to the Majorana nature of neutrino or/and different flavor prescriptions for the RH components of neutrinos and charged leptons. Correspondingly, different Higgs multiplets (flavons) participate in generation of their masses.

However, no convincing realization of this program has been proposed so far, although the simplest possibilities have been systematically checked. Specific models are based on many *ad hoc* assumptions, require introduction of auxiliary symmetries and new parameters, *etc.*, and usually do not lead to testable predictions. Already this posed doubts in the approach and new experimental results reinforced them. In this situation:

1. One can further follow the approach (1 - 3) exploring various possibilities to accommodate recent experimental results: Introduce large corrections from the charged lepton sector which do not obey the symmetry G_ν , or break G_ν in the neutrino sector immediately: $G_\nu \rightarrow 1_\nu$ [37].
2. One can still use the approach (1 - 3) considering “discrete symmetries without TBM” [38]. The “symmetry building” can be performed starting from symmetries of neutrino and charged lepton mass matrices in the mass basis. In the simplest version this leads [39] to

the von Dyck groups, $G_f = D(2, m, p)$ (m, p are integers) which include A_4, S_4, A_5 , and gives two relations between the elements of mixing matrix. This fixes 2 out of 4 mixing parameters. The relations are for one of the column (j) of the mixing matrix:

$$|U_{\beta j}|^2 = |U_{\gamma j}|^2, \quad |U_{\alpha j}|^2 = \frac{1-a}{4 \sin^2(\pi k/m)}, \quad (3)$$

where $\alpha \neq \beta \neq \gamma$ are the flavor indices $j = 1, 2, 3$; $k \leq m$ is integer and a is determined from the conditions

$$\lambda_i^3 + a\lambda_i^2 - a^* \lambda_i - 1 = 0, \quad \lambda_i^p = 1. \quad (4)$$

Here $i = 1, 2, 3$. The most interesting possibility is $p = 4, m = 3$ which corresponds to S_4 group. In this case $a = -1$ and for $j = 1$ one has, e.g., $|U_{e1}|^2 = 2/3$. For the best fit values of θ_{13} and θ_{23} one predicts then $\delta_{CP} = 103^\circ$. Without simplifications the approach can lead to 4 relations thus fixing the mixing parameters completely.

In view of the fact that also 1-2 mixing deviates from the TBM value one can consider the $\nu_\mu - \nu_\tau$ symmetry only, which would imply the equalities $\sin^2 \theta_{13} = d_{23} = 0$. Violation of this symmetry leads generically to related non-zero values of these parameters. If violation has “universal” character such that $m_{\mu\mu} - m_{\tau\tau} \approx m_{e\mu} - m_{\mu\tau}$ one obtains

$$\sin^2 \theta_{13} = \frac{1}{2} \cos^2 2\theta_{23} = 2d_{23}^2 \approx 0.022 \quad (5)$$

in perfect agreement with measurement.

3. One can abandon both the symmetry approach (1 - 2) and TBM. The Quark-lepton complementarity (QLC) is another realization of special zero order structure. QLC predicted [40]

$$\sin \theta_{13} \approx \frac{1}{\sqrt{2}} \sin \theta_C (1 - V_{cb} \cos \delta) - V_{ub} \approx \frac{\theta_C}{\sqrt{2}} \quad (6)$$

where θ_C is the Cabibbo angle. This prediction is essentially result of permutation of the matrices of maximal 2-3 rotation and 1-2 rotation on the Cabibbo angle:

$$U_{12}(\theta_C) U_{23}\left(\frac{\pi}{4}\right) \approx U_{23}\left(\frac{\pi}{4}\right) U_{13}\left(\frac{\theta_C}{\sqrt{2}}\right) U_{12}\left(\frac{\theta_C}{\sqrt{2}}\right).$$

(Such a permutation is needed to reduce the PMNS matrix to the standard. The simplest origin of the above structure is the following: In certain basis the matrix of up-quarks is diagonal, so that $V_u = I$, whereas diagonalization of the neutrino mass matrix gives the bi-maximal mixing $U_\nu = U_{12}(\pi/4) U_{23}(\pi/4)$. The latter may follow from the see-saw mechanism of neutrino mass generation. For the charged leptons and down-type quarks:

$U_l \approx V_d = V_{CKM}$ due to Grand Unification or the same horizontal symmetry. As a result:

$$\begin{aligned} U_{PMNS} &= U_l^\dagger U_\nu = V_{CKM}^\dagger U_{bm}, \\ V_{quarks} &= V_u^\dagger V_d = V_{CKM}. \end{aligned} \quad (7)$$

Taking in the PMNS mixing matrix $V_{CKM}^\dagger \approx U_{12}(\theta_C)^\dagger$ we obtain the required mixing structure and $\sin^2 \theta_{13} = 0.5 \sin^2 \theta_C$ [40, 41].

In the QLC framework the exact bimaximal mixing leads to deviation of the 2-3 mixing from maximal: $d_{23} \approx \cos \theta_C V_{cb} \cos \delta + 0.5 \sin^2 \theta_C$ which is about 0.06 for $\delta = 0$. Some corrections to the above picture may be needed. If neutrino mixing deviates from the bi-maximal one: $\sin \theta_{13} \approx \sin \theta_{23} \sin \theta_C$.

Also the QLC implies special structure for neutrinos which gives the bi-maximal mixing. The latter can be a consequence of certain symmetry of the RH neutrino mass matrix.

The weak complementarity [42] or Cabibbo “haze” [43] essentially mean that there are corrections to zero order structure of mixing matrix of the order of θ_C . E.g. the deviation of 2-3 mixing from maximal is of this order. The ratio of masses can also be determined by θ_C . The corrections are feature of the flavor physics and they do not imply quark-lepton symmetry and unification. No exact prediction for the angles can be done in this context.

The self-complementarity [44] is purely leptonic relation

$$\theta_{12} + \theta_{13} = \theta_{23}. \quad (8)$$

It is also reproduced by QLC.

4. One can reconsider the quark-lepton universality which means that there is nothing special in the lepton sector (apart from seesaw) and the Dirac leptonic mass matrices are organized in the same way as the quark mass matrices. Prediction for the 1-3 mixing was obtained from “naturalness” of mass matrix [45]:

$$\sin^2 \theta_{13} = A \frac{\Delta m_{21}^2}{\Delta m_{32}^2}, \quad (9)$$

where $A = 0.78$ ($\sim 1 - \sin \theta_C$?) for the best fit value of θ_{13} . This relation follows from the fact that there are two large mixing connecting neighboring generations, and from the following two assumptions: (i) NH, (ii) absence of fine tuning between different elements of the mass matrix (e.g., $m_{e\mu} = m_{e\tau}$). There are also models where the relation (9) is a consequence of certain symmetry [46].

A kind of Fritzsch ansatz can be used for the lepton Dirac mass matrices. If all RH neutrino masses are

equal each other, this leads via seesaw to the normal mass hierarchy and correct value of 1-3 mixing [47].

Another indication of the universality is that equalities of the same type

$$\theta_{13} \approx \frac{1}{2}\theta_{12}\theta_{23} \text{ and } V_{ub} \approx \frac{1}{2}V_{us}V_{cb}$$

are satisfied in the lepton and quark sectors.

5. Completely opposite approach is the mixing anarchy, in which mixing angles appear as random numbers [48]. At 1σ level it gives $\sin^2 \theta_{13} > 0.025$. Fit with anarchy can also be considered as a test of complexity behind neutrino mass and mixing.

Thus, the observed value of 1-3 mixing is reproduced in various ways (5), (6), (8), (9), which have completely different implications for fundamental theory.

Predictions for unknown yet quantities can be obtained reducing number of parameters and structures of a model applying the Occam's razor [19]. Good to know, though, where to cut: at the level of parameters, or assumptions, or principles. Usually reduction of number of parameters is achieved by prize of increasing number of assumptions. For instance, it was assumed in [19] that only two RH neutrinos are involved in seesaw, and there are two zeros in the Dirac mass matrix. This leads to phenomenologically viable scenario for the inverted mass hierarchy with its generic prediction for the Majorana mass $m_{ee} \approx 0.05$ eV and the CP-phase $\delta_{CP} \approx \pi/2$ [19].

Another realization of the Occam's razor, applied to number of assumptions is the following. (i) The high mass scale seesaw mechanism, probably associated to GUT scale, with 3 RH neutrinos explains smallness of neutrino mass. (ii) The Dirac Yukawa couplings in quark and lepton sectors are similar. (iii) The same seesaw is responsible for enhancement of lepton mixing. So, difference of the quark and lepton mixings is due to seesaw. (iv) Flavor structure originates from physics at very high scale, probably above GUT. The RH neutrino mass matrix may have a special structure. Some hidden sector may exist at GUT or/and Planck scales.

After many recent speculations, are we back to good old picture? Plausible scenario is SO(10) GUT with hidden sector (order of hundred SO(10) singlets). Fermionic components of this sector can mix with neutrinos only [49]. It is this mixing that produces all the observable differences in quark and lepton sectors: (i) smallness of neutrino mass via the double seesaw; (ii) required scale of the RH neutrinos; (iii) enhancement of lepton mixing; (iv) certain relations between mass matrix elements via symmetries in the hidden sector; (v)

the bi-maximal mixing required by QLC; (vi) it can also produce randomness (anarchy) in the lepton mixing.

Another possibility is additional vector-like GUT multiplets whose components mix with the components of the chiral multiplets.

Whole the beauty of the usual seesaw is that it explains smallness of neutrino mass by existence of a high mass scale without recurring to suppressed Yukawa couplings. The TeV-scale versions of seesaw which can be tested at LHC require both new mass scale and smallness (or fine tuning) of the neutrino Yukawa couplings. These are kind of “searches under a lamp” without serious motivations beside a possibility to test it at LHC. One exception is scenario which explains the Heidelberg-Moscow result on $\beta\beta 0\nu$ decay by exchange of new particles of the L-R symmetric models (RH neutrinos, double charged Higgs bosons) [50].

Anyway, according to [51] LHC can detect seesaw “messengers” with masses below TeV: RH neutrinos of the seesaw type I with $M_N \sim 10 - 400$ GeV, and M_N up to 400 GeV in the L-R models; Higgs triplet of the seesaw type-II with $M_\Delta = 400 - 1000$ GeV; the RH neutrino of the seesaw type-III with mass up to 800 GeV.

Phenomenologically viable alternative elaborated by the Occam's razor is the ν MSM [52] with the keV - GeV scale seesaw.

Different approach to understand neutrino mass and mixing can be developed assuming that Yukawa couplings have dynamical origin being identified with VEV's of scalar fields - flavons [53]. The flavons transform under certain representations of the flavor group. Values of VEV's, and consequently, Yukawa couplings are fixed by minimization of the potential of flavons fields which is invariant under the flavor group. In this context (as it was illustrated by a toy model) one can understand difference of mixings in quark and lepton sectors employing the Majorana nature of neutrinos.

The hope was (as before) that studies of neutrinos will uncover something exiting which shed light on other problems of particle physics, astrophysics and Cosmology. Special values of lepton mixing with certain flavor symmetries behind, indeed, supported the expectation. However, recent developments put some doubts and it seems we return back from “special to normal”.

6. Sterile neutrinos: challenge and opportunity

“Maiko-san session” gave another sign. Recall that in the beginning three Maiko-san's dressed in colorful kimonos were dancing. Then we saw two Maiko-sans in black and white. Then all five were dancing. A sign

of $3 + 2$! However, in the interpretation part only three Maiko-sans showed up and senior lady was giving explanations. I am not sure that this is $3 + 1$.

In any case possible existence of sterile neutrinos is the challenge for everything: theory, phenomenology and experiment.

Theory: corrections to mass eigenvalues and mixing angles of active neutrinos from the mixing with eV scale states required by LSND are, in general, of the order one. They change structure and symmetries of the mass matrix of active components significantly, unless special conditions are imposed on the active-sterile mixing (see [54]). On the other hand this mixing can be used to explain certain properties of active neutrinos, e.g., it can enhance mixing between active neutrino components. So, without clarification of existence of the eV steriles our further progress in understanding neutrino mass and mixing is almost impossible.

Phenomenology: the eV steriles are important for solar, atmospheric and supernova neutrinos, for beta and double beta decays, for reactor and accelerator neutrinos, for cosmology (nucleosynthesis, extra radiation of the universe, structure formation) [55]. They can affect searches of the dark matter.

Experiment: situation is rather controversial. MiniBooNE confirms LSND: similar excess is observed both in neutrino and antineutrino channels. But good description of the energy dependence of the MiniBooNE excesses would be possible with two steriles and CP-violation [56]. At the same time new contributions to the appearance signals have been revealed which reduces significance of the LSND (and also MiniBooNE) excess [57]. The issues of the cross-section and energy reconstruction are under discussion. With negative MINOS searches for steriles [58] tension between the disappearance data and the appearance LSND-MiniBooNE signals further strengthened [59]. According to fig. 4 there is a small region around 1 eV^2 in which 3σ allowed and preferred regions overlap. With only one sterile fit of the energy spectra is far from being perfect.

Situation with the reactor and Gallium anomalies [60] is less dramatic. There is new bound obtained from joint fit of solar, KamLAND, Daya-Bay and Reno experiments: $\sin^2 2\theta_{es} < 0.2$ [59]. Global fit gives the best value $\Delta m^2 = 1.78 \text{ eV}^2$ which, however, is not consistent with ν_μ -oscillation results. Analysis of cosmological data in terms of ΛCDM leads to the bound $\Delta m_{41}^2 < 0.25 \text{ eV}^2$ for 1 sterile [61].

For effective number of neutrino species (extra radiation in the Universe) one has $N_\nu = 3.5 \pm 0.3, 1\sigma$ [62].

IceCube is ideal detector of the eV mass steriles [63]. For $\Delta m^2 \sim 1 \text{ eV}^2$ the MSW resonance is realized in the

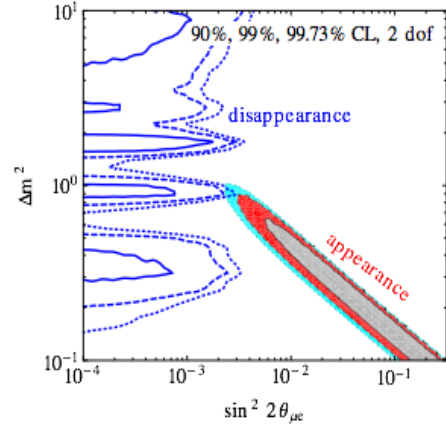


Figure 4: Allowed and excluded regions of parameters of sterile neutrino $\Delta m_{41}^2 - \sin^2 2\theta_{e\mu}$ within $3 + 1$ scheme. From [59].

$\bar{\nu}_\mu - \bar{\nu}_s$ channel in matter of the Earth at neutrino energies $\sim 1 \text{ TeV}$. The resonant enhancement of oscillations in this energy range leads to distortion of the zenith angle distribution of the μ -like atmospheric neutrino events. The effect (which depends also on admixture $U_{\tau 4}$) is of the order $10 - 20\%$. Statistics (few $\times 10^3$ events in each of 20 bins) is not a problem. Once systematic uncertainties are understood, IceCube will provide critical test of existence of the eV steriles.

7. Neutrinos and the Dark universe

There are various ideas about how neutrino can be related to the dark sector of the Universe: dark matter and dark energy. Direct connection: new neutrino states with the keV scale mass can compose the worm DM. Indirect connections: the same flavor symmetries which are responsible for mixing pattern or/and for smallness of neutrino mass can ensure stability of DM. New particles involved in mechanisms of neutrino mass generation can play the role of DM.

Neutrino mass generation can be related via the see-saw with inflation, leptogenesis and production of the Dark matter particles: “everything in one” [scenario] [64]. Indeed, the inflaton field can act (in SUSY context) as a driving field which leads to VEV of scalar S that breaks the B-L symmetry: S couples with the RH neutrinos, N_i , and generate their masses. Then evolution can proceed in the following way: Reheating occurs via decays of $S \rightarrow NN$ and subsequent decay of RH neutrinos in to SM particles: $N \rightarrow lH$. The decay of the lightest of them, N_1 , with mass M_1 produced the

lepton asymmetry which then converted to the baryon asymmetry. The reheating temperature $T_{RH} = T_{RH}(M_1)$ determines the relic density of the thermally produced gravitinos which play the role of the DM particles. The successful scenario is realized for $\nu_{B-L} \sim 5 \cdot 10^{15}$ GeV, $M_1 \sim 10^{11}$ GeV and $T_{RH} \sim 10^{10}$ GeV and the effective light neutrino mass $\tilde{m}_1 \sim 0.04$ eV. The latter implies the bound on mass of gravitino $m_g > 10$ GeV.

Hints of extra radiation are another driving force of developments. Still BBN prefers $N_\nu > 3$, although recent determination of the deuterium abundance [65] results in $N_\nu = 3.0 \pm 0.5$ [15]. The CMB data give stronger indications. With CMB, higher value of the Hubble constant drives N_ν to 3.5–3.7. The highest value of ΔN_ν follows from analysis which includes data from WMAP, ACBAR and BAO. High values of H_0 tend to increase ΔN_ν , whereas ACT data decreases it [66]. The Planck cosmological data are expected to be sensitive to $\Delta N_\nu \approx 0.2$.

Another possible connection: the neutrino velocity can be affected by the dark sector of the Universe. One still can explore whether, e.g., “short cut in extra dimensions” can be the reason of bad fiber connection in the OPERA experiments. Instead, I will make few statements which could be of some relevance.

1. Neutrino is the lightest massive particle we know. Therefore for available energies its velocity can be the closest one to velocity of light.

2. If measurements of neutrino velocity are continued, an important question is what can be checked at the achievable level of sensitivity? Numerous papers issued recently (worth to analyze part of them) contain some answers to this.

3. Fundamental symmetries can be violated effectively due to interactions with some background. Recall, e.g., that CPT is violated in oscillations in usual medium.

4. In this connection, do we know well the background? Do we know everything about dark sector of the Universe?

5. The proposal of neutrino oscillations by B. Pontecorvo was motivated by rumor that Ray Davis saw effect in the Cl-Ar detector from atomic reactor [67].

8. Outlook

1. To a large extend future developments in theory and phenomenology will be driven by new experimental highlights: rather large value of $\sin^2 \theta_{13}$ and indication of significant deviation of the 2-3 mixing from maximal. Global fit gives first glimpses on to the CP-phase, and it is the time to make predictions for δ_{CP} .

2. Interesting developments can be related to the solar neutrinos, where absence of the spectral upturn is further confirmed. That can be related to some tension with KamLAND value of Δm_{21}^2 and a bit higher value of the day-night asymmetry reported by SK. Is this just accidental, statistical fluctuation or accumulation of results which testify for new physics, e.g., new very light steriles or new interactions?

3. The race for mass hierarchy has started. Although phenomenology of different hierarchies is well elaborated still some more ideas to identify ordering may appear. It may happen that new developments will be associated to huge atmospheric neutrino detectors (HAND's) with low (~ 1 GeV) energy threshold. HAND's may perform good measurements of θ_{23} . Sensitive search of steriles with IceCube will be realized. Once hierarchy is established one can explore in more details searches of CP-violation. A possibility to determine CP-violation with HANDs is challenging but not completely excluded.

4. In perspective, the threshold of the huge detectors can be further reduced down to 10^{-2} GeV (MICA). Physics potential of this type of detection still should be explored.

5. Presentation of the two IceCube candidates of cosmic neutrinos (cascades) was rather modest. The discovery (if confirmed) will have enormous impact on the field and already these two events trigger various speculations. Implications of the null IceCube result, in particular, for searches of neutrinos associated to GRB will be in the center of studies.

6. The tribimaximal mixing is further disfavored. Still TBM can be considered as the lowest order structure which requires significant corrections. In view of the fact that no convincing model for that has been proposed, the paradigm may change. E.g. one can abandon TBM and use some other zero order structure, or pursue the same flavor symmetry approach without TBM.

The relation $\theta_{13} \sim \theta_C / \sqrt{2}$ predicted in the context of QLC, is in a good agreement with recent measurements. Is this accidental?

It could be certain shift in our understanding of mass and mixing from “special to normal”, and things with leptons become closer to quarks. Probably the Dirac Yukawa structures are similar in both cases, symmetry (if exists) is the same for quarks and leptons, there is no special symmetry in the lepton sector. Smallness of neutrino mass and large lepton mixing originate from the same seesaw. In this connection we expect normal mass hierarchy, mass matrices with flavor ordering, high scale seesaw, enhancement of mixing. GUT embedding may require new elements, in particular, mixing of neu-

trinos with new singlets of the gauge symmetry group.

7. Steriles are challenge for neutrino physics. There are controversial experimental evidences, tension with cosmology, tension between the appearance and disappearance data, puzzling theoretical situation. On the other hand, steriles have rich phenomenology and their existence opens up new ways to understand the observed mixing and masses. New searches of steriles with atmospheric neutrinos will be performed using Ice-Cube, DeepCore, and dedicated source, reactor and accelerator experiments.

8. Neutrinos and dark Universe will continue to be one of exiting areas of research. Further studies of possible connections may lead to breakthrough both in particle physics and Cosmology.

References

- [1] G. L. Fogli, in these Proceedings, G. L. Fogli, E. Lisi, A. Marone, D. Montanino, A. Palazzo and A. M. Rotunno, *Phys. Rev. D* **86**, 013012 (2012) [arXiv:1205.5254 [hep-ph]].
- [2] H. Minakata, in these Proceedings.
- [3] C. Wagner, L. Verde and R. Jimenez, arXiv:1203.5342 [astro-ph.CO] and references therein; A. C. Hall and A. Challinor, arXiv:1205.6172 [astro-ph.CO].
- [4] S. T. Petcov and M. Piai, *Phys. Lett. B* **533**, 94 (2002); J. Learned, S. T. Dye, S. Pakvasa and R. C. Svoboda, *Phys. Rev. D* **78**, 071302 (2008); P. Ghoshal and S. T. Petcov, arXiv:1208.6473 [hep-ph], and references therein.
- [5] S. Choubey, in these Proceedings.
- [6] R. B. Patterson [NOvA Collaboration], in these Proceedings. [arXiv:1209.0716 [hep-ex]].
- [7] J. Tang and W. Winter, *JHEP* **1202**, 028 (2012).
- [8] A. S. Dighe and A. Y. Smirnov, *Phys. Rev. D* **62**, 033007 (2000); C. Lunardini and A. Y. Smirnov, *Nucl. Phys. B* **616**, 307 (2001); *JCAP* **0306**, 009 (2003).
- [9] S. Razzaque and A. Y. Smirnov, *Phys. Rev. D* **85**, 093010 (2012) [arXiv:1203.5406 [hep-ph]].
- [10] E. K. Akhmedov, S. Razzaque and A. Y. Smirnov, arXiv:1205.7071 [hep-ph].
- [11] M. Auger *et al.* [EXO Collaboration], *Phys. Rev. Lett.* **109**, 032505 (2012) [arXiv:1205.5608 [hep-ex]].
- [12] H. V. Klapdor-Kleingrothaus and I. V. Krivosheina, *Mod. Phys. Lett. A* **21**, 1547 (2006).
- [13] P. Grabmayr, in these Proceedings.
- [14] S. M. Bilenky and C. Giunti, *Mod. Phys. Lett. A* **27**, 1230015 (2012) [arXiv:1203.5250 [hep-ph]], for earlier analysis see F. Vissani, *JHEP* **9906** (1999) 022 [hep-ph/9906525].
- [15] L. Verde, in these Proceedings, M. Moresco, *et al.*, *JCAP* **1207**, 053 (2012) [arXiv:1201.6658 [astro-ph.CO]]. R. de Putter, *et al.*, arXiv:1201.1909 [astro-ph.CO].
- [16] R. Nichol, in these Proceedings.
- [17] D. J. Koskinen, *Mod. Phys. Lett. A* **26**, 2899 (2011).
- [18] Y. Farzan and A. Y. Smirnov, *Phys. Rev. D* **65**, 113001 (2002).
- [19] T. Yanagida, in these Proceedings; K. Harigaya, M. Ibe and T. T. Yanagida, *Phys. Rev. D* **86**, 013002 (2012) [arXiv:1205.2198 [hep-ph]].
- [20] M. Smy, in these Proceedings.
- [21] B. Aharmim *et al.* [SNO Collaboration], arXiv:1109.0763.
- [22] M. Pallavicini, in these Proceedings.
- [23] S. Abe *et al.* [KamLAND Collaboration], *Phys. Rev. C* **84**, 035804 (2011) [arXiv:1106.0861 [hep-ex]].
- [24] P. C. de Holanda and A. Y. Smirnov, *Phys. Rev. D* **83**, 113011 (2011) [arXiv:1012.5627 [hep-ph]].
- [25] A. Palazzo, *Phys. Rev. D* **83**, 101701 (2011) [arXiv:1101.3875].
- [26] A. Serenelli, in these Proceedings.
- [27] A. McDonald, in these Proceedings.
- [28] G. Sullivan, in these Proceedings.
- [29] P. Coyle, in these Proceedings, S. Adrian-Martinez *et al.* [ANTARES Collaboration], *Phys. Lett. B* **714**, 224 (2012).
- [30] B. Balantekin, in these Proceedings.
- [31] A. B. Balantekin, arXiv:1208.0386 [nucl-th], Y. Pehlivan, A. B. Balantekin, T. Kajino and T. Yoshida, *Phys. Rev. D* **84**, 065008 (2011) [arXiv:1105.1182 [astro-ph.CO]]. A. B. Balantekin, arXiv:1111.2282 [astro-ph.SR].
- [32] see e.g., N. Saviano, S. Chakraborty, T. Fischer and A. Mirizzi, *Phys. Rev. D* **85**, 113002 (2012) [arXiv:1203.1484 [hep-ph]], and references therein.
- [33] S. Pastor, in these Proceedings.
- [34] M. Ahlers, in these Proceedings.
- [35] A. Ishihara, in these Proceedings.
- [36] C. S. Lam, arXiv:0907.2206 [hep-ph], *Phys. Rev. Lett.* **101**, 121602 (2008) [arXiv:0804.2622 [hep-ph]]. *Phys. Rev. D* **78**, 073015 (2008) [arXiv:0809.1185 [hep-ph]], W. Grimus, L. Lavoura and P. O. Ludl, *J. Phys. G G* **36**, 115007 (2009) [arXiv:0906.2689 [hep-ph]].
- [37] G. Altarelli, F. Feruglio, L. Merlo and E. Stamou, *JHEP* **1208**, 021 (2012) [arXiv:1205.4670 [hep-ph]].
- [38] Shao-Feng Ge, *et al.* *Phys. Rev. Lett.* **108** (2012) 041801 arXiv:1108.0964 [hep-ph], *Phys. Lett. B* **702** (2011) 220–223 arXiv:1104.0602 [hep-ph], Hong-Jian He, *et al.* arXiv:1203.2908 [hep-ph].
- [39] D. Hernandez and A. Y. Smirnov, arXiv:1204.0445 [hep-ph].
- [40] H. Minakata and A. Y. Smirnov, *Phys. Rev. D* **70**, 073009 (2004), M. A. Schmidt and A. Y. Smirnov, *Phys. Rev. D* **74**, 113003 (2006).
- [41] S. Antusch, in these Proceedings.
- [42] L. Merlo, *Acta Phys. Polon. B* **40**, 3179 (2009).
- [43] A. Datta, L. Everett and P. Ramond, *Phys. Lett. B* **620**, 42 (2005) [hep-ph/0503222].
- [44] X. Zhang and B. -Q. Ma, *Phys. Lett. B* **710**, 630 (2012).
- [45] E. K. Akhmedov, G. C. Branco and M. N. Rebelo, *Phys. Rev. Lett.* **84**, 3535 (2000) [hep-ph/9912205]; M. Frigerio and E. Ma, W. Rodejohann, M. Tanimoto and A. Watanabe, *Phys. Lett. B* **710**, 636 (2012) [arXiv:1201.4936 [hep-ph]].
- [46] M. Frigerio and E. Ma, *Phys. Rev. D* **76**, 096007 (2007).
- [47] M. Fukugita, Y. Shimizu, M. Tanimoto and T. T. Yanagida, *Phys. Lett. B* **716**, 294 (2012) [arXiv:1204.2389 [hep-ph]].
- [48] A. de Gouvea and H. Murayama, arXiv:1204.1249 [hep-ph]; G. Altarelli, F. Feruglio, I. Masina and L. Merlo, arXiv:1207.0587 [hep-ph].
- [49] W. Buchmuller, *et al.* *Phys. Rev. Lett.* **99**, 021601 (2007) [hep-ph/0703078 [HEP-PH]]; B. Feldstein and W. Klemm, *Phys. Rev. D* **85**, 053007 (2012) [arXiv:1111.6690 [hep-ph]].
- [50] M. Nemevsek, F. Nesti, G. Senjanovic and V. Tello, arXiv:1112.3061 [hep-ph].
- [51] Tao Han, in these Proceedings.
- [52] L. Canetti, M. Drewes, T. Frossard and M. Shaposhnikov, arXiv:1208.4607 [hep-ph].
- [53] B. Gavela, in these Proceedings.
- [54] J. Barry, W. Rodejohann and H. Zhang, *JHEP* **1107**, 091 (2011).
- [55] K. N. Abazajian, *et al.*, arXiv:1204.5379 [hep-ph].
- [56] C. Polly, in these Proceedings; A. A. Aguilar-Arevalo *et al.* [MiniBooNE Collaboration], arXiv:1207.4809 [hep-ex].
- [57] D. Dedovich and A. Zhemchugov, *Mod. Phys. Lett. A* **27**,

- 1230012 (2012).
- [58] A. B. Sousa [MINOS Collaboration], arXiv:1110.3455 [hep-ex]; P. Adamson *et al.* [MINOS Collaboration], Phys. Rev. Lett. **107**, 011802 (2011) [arXiv:1104.3922 [hep-ex]].
- [59] T. Schwetz, in these Proceedings.
- [60] T. Lasserre, in these Proceedings.
- [61] M. Archidiacono, N. Fornengo, C. Giunti and A. Melchiorri, arXiv:1207.6515 [astro-ph.CO].
- [62] E. Giusarma, M. Archidiacono, R. de Putter, A. Melchiorri and O. Mena, Phys. Rev. D **85**, 083522 (2012) [arXiv:1112.4661].
- [63] H. Nunokawa, et al. Phys.Lett. B562 (2003) 279 (2003); S. Choubey, JHEP 0712 (2007) 014 arXiv:0709.1937; S. Razzaque and A. Y. Smirnov, JHEP **1107**, 084 (2011) [arXiv:1104.1390 [hep-ph]]; A. Esmaili, F. Halzen and O. L. G. Peres, arXiv:1206.6903 [hep-ph].
- [64] W. Buchmuller, in these Proceedings, W. Buchmuller, V. Domcke and K. Schmitz, Nucl. Phys. B **862**, 587 (2012).
- [65] M. Pettini and R. Cooke, arXiv:1205.3785 [astro-ph.CO].
- [66] E. Calabrese, M. Archidiacono, A. Melchiorri and B. Ratra, Phys. Rev. D **86**, 043520 (2012).
- [67] S. M. Bilenky, physics/0603039.